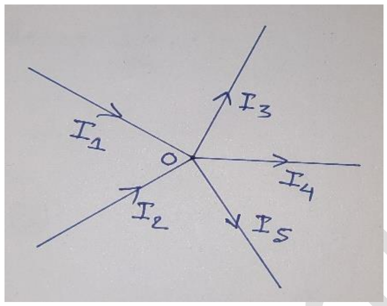
Electrical circuit

Kirchhoff's laws

Kirchhoff's first law / Kirchhoff's junction law / Kirchhoff's current (KCL):

It states that, "The algebraic sum of all currents meeting at a junction point in an electrical circuit is zero." i.e.

$$\sum I = 0$$



Sign of Convention
According to sign convention, current entering to the junction point is taken as positive and the current leaving through the junction point is taken as negative.

i.e. I_1 and I_2 are positive but I_3, I_4 and I_5 are negative.

From Kirchhoff's first law, we can write

$$\sum I = 0$$

Or, $I_1 + I_2 + (-I_3) + (-I_4) + (-I_5) = 0$
Or, $I_1 + I_2 = I_3 + I_4 + I_5$
From above result, Kirchhoff's first law also can be stated as "The sum of all currents entering to the junction point is equal to the sum of all currents leaving through the junction point". This law is also known as current law or junction law.
The principle of Kirchhoff's first law is 'conservation of charge'.

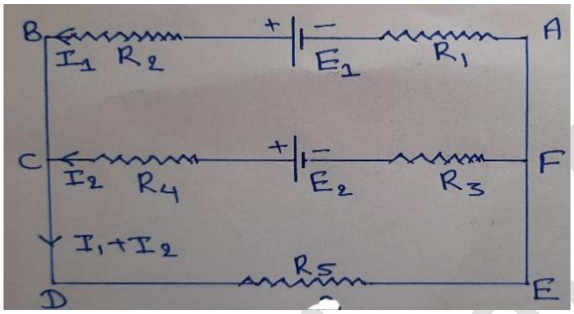
Kirchhoff's second law / Kirchhoff's loop law / Kirchhoff's voltage law (KVL):

It states that, "In any close loop of electrical circuit, the algebraic sum of emf of all batteries is equal to algebraic sum of product of current and resistance through which it flows."
i.e. $\sum E = \sum IR$

Sign convention

1. In any closed loop, if the direction of loop is from negative terminal to positive terminal of a battery, the emf is taken as positive and if the direction loop is from positive terminal to negative terminal, the emf is taken as negative.
2 In any closed loop, if the direction of loop is in the direction of flow of current, the product of current and resistance (IR) is taken as positive and if the direction of loop is in the direction opposite to the flow of current, the product of current and resistance is taken as negative.

Explanation



Applying Kirchhoff's voltage law (KVL) in closed loop ABCFA,
According to sign convention, E_1 is positive but E_2 is negative. Also, I_1R_1 and I_1R_2 are positive but I_2R_3 and I_2R_4 are negative.
Now we can write,
$E_1 - E_2 = I_1R_1 + I_1R_2 - I_2R_3 - I_2R_4$
Applying Kirchhoff's voltage law (KVL) in closed loop CDEFC,
According to sign convention, E_2 is positive, I_2R_3, I_2R_4 and $(I_1+I_2)R_5$ all are Positive. Now, we can write,
$E_2 = I_2R_3 + I_2R_4 + (I_1 + I_2)R_5$

Wheatstone bridge:
It is an electrical circuit which is used to measure value of unknown resistance.

Principle: "A Wheatstone bridge works on the principle of null deflection.i.e.no current flows through the galvanometer."

The balanced condition for Wheatstone bridge is

For, $I_g = 0$, $\frac{P}{Q} = \frac{X}{R}$

Construction:

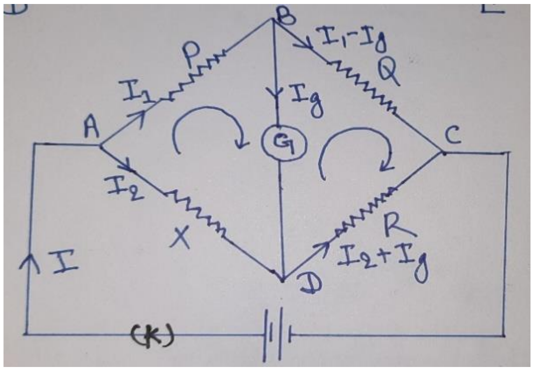


Fig: Wheatstone bridge

A Wheatstone bridge consists of four resistance P, Q, R and X placed in the form of quadrilateral ABCD as shown in figure above. The resistance P and Q are fixed value resistance. R is variable resistance which is used to change the value of current and X is unknown resistance whose value is to be determined. A sensitive galvanometer is connected in between B and D and the point A and C are connected with positive and negative terminal of a battery through a key shown in figure above.

Working

When key is closed, an amount of current I starts to flow. From point A, the current I gets divided into I_1 and I_2 and flows through resistance P and X respectively. Let I_g be the amount of current flowing through galvanometer (B to D). Then $(I_1 - I_g)$ and $(I_2 + I_g)$ be the amount of current flow through resistance Q and R respectively.

On applying Kirchhoff's 2nd law in closed loop ABDA,

$I_1P + I_g \times G - I_2X = 0$ ...(i)

On applying Kirchhoff's 2nd law in closed loop BCDB,

$(I_1 - I_g)Q - (I_2 + I_g)R - I_gG = 0$ ....(ii)

Wheatstone bridge works on the principle of null deflection i.e. no amount of current flows through the galvanometer. i.e. $I_g = 0$

Now, equation (i) and (ii) become,

$I_1P - I_2X = 0$

Or, $I_1P = I_2X$ ...(iii)

and $I_1Q - I_2R = 0$

Or, $I_1Q = I_2R$ ....(iv)

Dividing equation (iii) by (iv), we get

$$\frac{I_1P}{I_1Q} = \frac{I_2X}{I_2R}$$

Or, $\frac{P}{Q} = \frac{X}{R}$ This is the balanced condition of Wheatstone Bridge

Also, $X = \frac{P}{Q}R$

Now, knowing the values of P, Q and R, the value of X can be determined.

Meter bridge:

It is an electrical circuit which is used to measure the value of unknown resistance. The wire used in this instrument is exactly one-meter that's why its name is Meter Bridge.

Principle

A meter bridge works upon the principle of balanced condition of Wheatstone bridge i.e. no amount of current flows through the galvanometer. The balanced condition of Wheatstone bridge is

$$\frac{P}{Q}=\frac{X}{R}\text{ ....(i)}$$

Construction

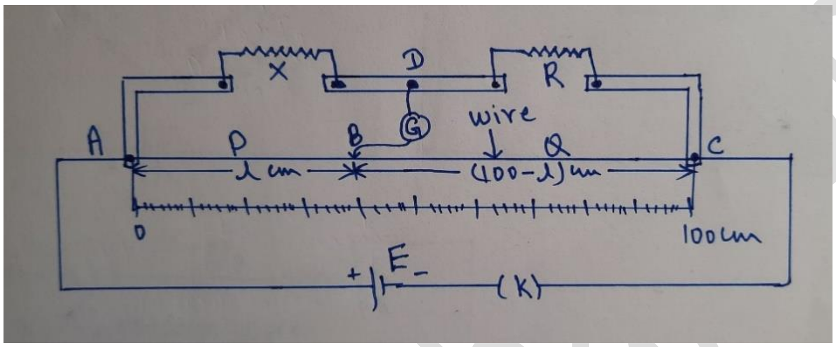


Fig: Determination of unknown resistance using meter bridge

A meter bridge consists of a uniform wire AC of length one meter (100cm) made up of manganin or constantan having high melting point and low value of temperature coefficient. The wire AC is stretched between two points on a wooden board provided with two L shaped copper stripe. Another copper stripe is fitted between two L shaped copper stripe providing with two gaps. A variable resistance R is placed in right gap and unknown resistance X is placed in left gap whose value is to be determined. One end of a sensitive galvanometer is connected at a point D and another end of galvanometer is connected with a jockey which can freely slide on the wire of meter bridge. The end A is connected with positive terminal and C is connected with negative terminal of a battery through a key (K). A meter scale is placed parallel to the length of wire.

Working

When key is closed, an amount of current starts to flow. When the Jockey is placed in contact with wire AC, the galvanometer shows deflection. On sliding the Jockey on the wire AC, suppose at any point B at a distance l from A, the galvanometer shows null deflection. Let AB = l cm and BC =(100 - l) cm.

Let resistance of a wire from A to B is P and from B to C is Q.

Since, we know that resistance of uniform wire is directly proportional to length,

i.e. $p \propto l$

Or, P = Kl..........(ii)

And, $Q \propto (100-l)$

Or, Q = k (100-l)......(iiii)

Dividing equation (ii) by (iii) we get,

$$\frac{P}{Q}=\frac{l}{(100-l)}$$

From equation (i),

$$\frac{X}{R}=\frac{l}{(100-l)}$$

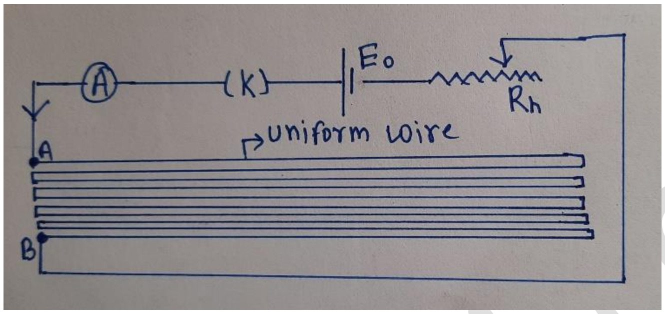
$$X=\frac{l}{(100-l)}R$$

This is the required expression to determine the value of unknown resistance.

Potentiometer

It is an electrical instrument which is used to measure emf of a cell, to compare emf of two cells and to determine internal resistance of a cell.

Construction:



Potentiometer consists of a uniform wire AB of length about 10m made up of constantan or manganin fixed on a wooden board. The end A is connected with positive terminal and B is connected with negative terminal of driver cell through a key, an ammeter and a variable resistance (Rheostat, R_h) as shown in figure above. The driver cell (accumulator) is used to supply constant amount of current and variable resistance is used to change the value of current. A meter scale is placed parallel to the length of wire.

Principle

The potentiometer works upon the principle that "The potential drop across any portion of the potentiometer wire is directly proportional to the length of that proportion. i.e.

$V \propto l$

Or, $V = kl$

i.e. potential gradient $\frac{V}{l}$ = constant.

Explanation:

Let 'a' be the area of cross section of potentiometer wire and its resistivity is ρ . Let us take a portion of the potentiometer wire of length 'l'. Now resistance of that portion is

$R = \rho \frac{l}{a}$

When key is closed and amount of current I starts to flow through the potentiometer wire. Now, the potential dropped across the portion of length l of resistance R is

$V = IR$

or, $V = I \times \rho \frac{l}{a}$

or, $V = k, l$ *(where* $k = \rho \frac{I}{a}$ *are constant)*

$\therefore V \propto l$

Applications of potentiometer:

1. Comparison of emf of two cells using potentiometer:

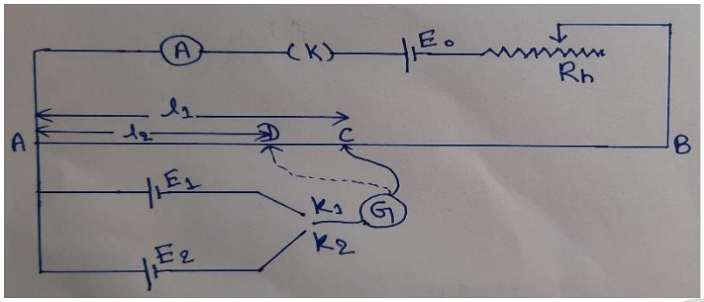


Fig: Comparison of emf of two cell using potentiometer

Let us consider a potentiometer of uniform wire AB of length about 10m. The end A of the wire is connected with positive terminal and end B is connected with negative terminal of driver cell through an ammeter, a key and a variable resistance (Rheostat, R_h) as shown in figure above. Let us consider two cell of emf E_1 and E_2, in which the value of $E_1$is known but E_2 is to be determined. Now, positive terminals of cell E_1 and E_2 are connected with a point A and negative terminals are connected with two-way key as shown in figure. One end of sensitive galvanometer is connected with two-way key and another end is connected with jockey which can freely slide on the wire of potentiometer.

When key K_1 is closed and K_2 is opened, on doing so the circuit will be open for cell E_2. On sliding the jockey on the potentiometer wire, let at any point C at distance l_1 from A, the galvanometer shows null deflection. Let V_{AC} be the potential across AC.

Then for $I_g = 0$ we can write,

$E_1 = V_{AC}$............(i)

According to the principle of potentiometer, we can write,

$V_{AC} \propto l_1$

Or, $V_{AC} = Kl_1$

Or, $E_1 = Kl_1$ ....(ii)

Similarly, when key K_2 is closed and K_1 is opened, on doing so the circuit will be open for cell E_1. On sliding the jockey on the potentiometer wire, let at any point D at distance l_2 from A, the galvanometer shows null deflection. Let V_{AD} be the potential across AD.

Then for $I_g = 0$ we can write,

$E_2 = V_{AD}$............(iii)

From the principle of potentiometer, we can write,

$V_{AD} \propto l_2$

Or, $V_{AD} = Kl_2$

Or, $E_2 = Kl_2$ ....(iv)

Dividing equation (ii) & (iv) we get,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

This is the required expression for comparison of emf of two cells using potentiometer.

(ii) Determination of internal resistance of a cell using potentiometer:

Let us consider a potentiometer of uniform wire AB of length about 10m. The end A of the wire is connected with positive terminal and end B is connected with negative terminal of driver cell through an ammeter, a key and a variable resistance (Rheostat, R_h) as shown in figure above.

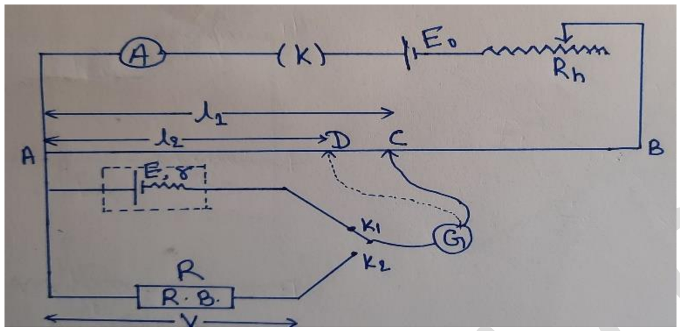


Fig: Determination of internal resistance of a cell using potentiometer

Let us consider an experimental cell of emf 'E' has an internal resistance 'r'. This value of internal resistance 'r' is to be determined and also let us take a known value of resistance R (using resistance box R.B.). The positive terminal of the cell and one end of 'R' are connected with point 'A' and negative terminal of cell and another end of 'R' are connected with one end of sensitive galvanometer through two way key K_1 and K_2 respectively. The other end of galvanometer is connected with a jockey which can freely slide on the wire of potentiometer.
When key K_1 is closed and K_2 is opened, on doing so the circuit will be open for R. On sliding the jockey on the potentiometer wire, let at any point C at distance l_1 from A, the galvanometer shows null deflection. Let V_{AC} be the potential across AC.
Then for $I_g = 0$ we can write,
$E = V_{AC}$............(i)
According to the principle of potentiometer, we can write,
$V_{AC} \propto l_1$
Or, $V_{AC} = Kl_1$
Or, $E = Kl_1$ ....(ii)
Similarly, when key K_1 and K_2 both are closed then on sliding the jockey on the potentiometer wire, let at any point D at distance l_2 from A, the galvanometer shows **null deflection**. Let V_{AD} be the potential across AD.
Then $I_g = 0$ but in this case, an amount of current I flows through the external resistance R and let V ve the potential difference across the resistor R then we can write,
$V = V_{AD}$............(iii)
From the principle of potentiometer, we can write,
$V_{AD} \propto l_2$
Or, $V_{AD} = Kl_2$
Or, $V = Kl_2$ ....(iv)

Dividing equation (ii) by (iv) we get,

$$\frac{E}{V} = \frac{l_1}{l_2}$$

Using circuit i.e. E = I (R+r), above equation becomes

or, $$\frac{I(R+r)}{IR} = \frac{l_1}{l_2}$$

or, $\frac{R}{R}+\frac{r}{R} = \frac{l_1}{l_2}$

or, $1+\frac{r}{R} = \frac{l_1}{l_2}$

or, $\frac{r}{R} = \frac{l_1}{l_2} - 1$

$\therefore\ r = \left(\frac{l_1 - l_2}{l_2}\right)R$

This is the required expression for determination of internal resistance using potentiometer.
Now, knowing the values of l_1, l_2 and R, the value of 'r' can be determined.

3. Determination (measurement) of emf of a cell using potentiometer:

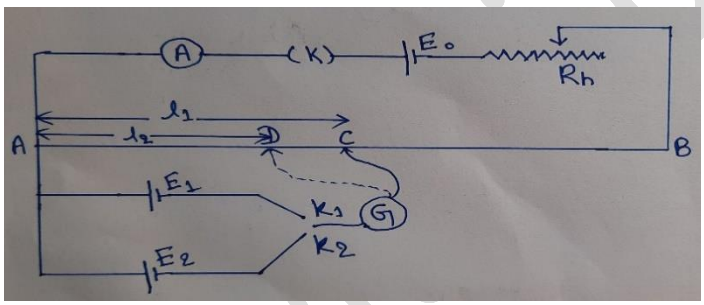


Fig: Determination of emf of a cell using potentiometer

Let us consider a potentiometer uniform wire AB of length about 10m. The end A of the wire is connected with positive terminal and end B is connected with negative terminal of driver cell through an ammeter, a key and a variable resistance (Rheostat, R_h) as shown in figure above. Let us consider two cell of emf E_1 (unknown and to be determined) and E_2 (known) , in which the value of $E_1$is known but E_2 is to be determined. Now, positive terminals of cell E_1 and E_2 are connected with a point A and negative terminals are connected with two-way key as shown in figure. One end of sensitive galvanometer is connected with two-way key and another end is connected with jockey which can freely slide on the wire of potentiometer.

When key K_1 is closed and K_2 is opened, on doing so the circuit will be open for cell E_2. On sliding the jockey on the potentiometer wire, let at any point C at distance l_1 from A, the galvanometer shows null deflection. Let V_{AC} be the potential across AC.

Then for $I_g = 0$ we can write,

$E_1 = V_{AC}$............(i)

According to the principle of potentiometer, we can write,

$V_{AC} \propto l_1$

Or, $V_{AC} = Kl_1$

Or, $E_1 = Kl_1$ ....(ii)

Similarly, when key K_2 is closed and K_1 is opened, on doing so the circuit will be open for cell E_1. On sliding the jockey on the potentiometer wire, let at any point D at distance l_2 from A, the galvanometer shows null deflection. Let V_{AD} be the potential across AD.

Then for $I_g = 0$ we can write,

$E_2 = V_{AD}$............(iii)

From the principle of potentiometer, we can write,

$V_{AD} \propto l_2$

Or, $V_{AD} = Kl_2$

Or, $E_2 = Kl_2$ ....(iv)

Dividing equation (ii) & (iv) we get,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$E_1 = \frac{l_1}{l_2} \times E_2$$

Knowing the values of l_1, l_2 and E_1, the value of E_2 can be determined.